Dr. Virendra Swarup Education Centre, Panki, Kanpur

Class:XII A/B **Subject- Mathematics (041)**

Holiday Home Work

General instruction: Do all the questions.

1) Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) ad (b + c) = bc (a +d). Check whether R is an equivalence relation on $\times N$.

2) Show that the relation R on **R** defined as R = {(a, b): a $\leq b$ } is reflexive and transitive but not Symmetric.

3) Show that the function $f: R \to R$ defined by $f(x) = 3x^3 + 5\ \forall x \in R$ is a bijection.

4) Let A = R-{2} and B = R-{1} $f: A \to B$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, show that f is bijective.

5) Check the injectivity and surjectivity for function defined by $f: z \to z\ f(x) = x^2 + x$.

6) Prove that the function $f: N \to N$defined by $f(x) = x^2 + x + 1$ is one-one but not onto.

7) Let S be a relation on the set R of all real numbers defined by $S = \{(a, b) \in R \times R: x^2 + y^2 = 25\}$ Prove that S is not an equivalence relation on R.

8) Find the domain of $f(x) = \sin^{-1}(2x - 3)$.

9) Find the domain of $f(x) = \sin^{-1} x + cosx$.

10) If x, y, z $x \in [-1,1]$ such that $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{-3\pi}{2}$, find the value of x^2+$y^2 + z^2$.

11) Find the principal value of $\sin^{-1}\left(\frac{-1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$.

12) Find the principal value of $cot[\sin^{-1}\{\cos(\tan^{-1} 1)\}]$.

13) Find the principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) - 2\sec^{-1}\left(2tan\frac{\pi}{6}\right)$.

14) Find the principal value of $\text{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right) + 2\cot^{-1}(-1)$.

15) If $\begin{bmatrix} x+1 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-1 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$, find the value of a, b, c, x, y and z.

16) Solve the matrix equation $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$.

17) If A = $\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, then find the matrix X of order 3× 2 such that 2A +3 X = 5B.

18) The monthly incomes of Aryan and Rahul are in the ratio 3: 4 and their monthly expenditures are in the ratio 5:7.If each saves Rs 1500 per month, find their monthly incomes using matrix method.

19) If A = $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, find x and y such that $(xI + yA)^2 = A$.

20) If AB = A and BA = B then show that $A^2 = A$, $B^2 = B$.

21) If A = [3 5] B = [7 3] ,then find a nonzero matrix C such that AC = BC.

22) If A = $\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, find $A^2 - 5A - 14I$.

23) If $f(x) = x^3 + 4x^2 - x$, find $f(A)$ where A = $\begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$.

24) If A = $\begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, Prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$ for every positive integer n.

25) Find the values of x, y, z if the matrix A = $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfy the equation$A^T A = I_3$.

26) Express the matrix A = $\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$as a sum of symmetric and skew – symmetric matrix.

P.T.O

27) Find the integral value of x, if $\begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix}$ = 28.

28) If the points (a_1, b_1) , (a_2, b_2) and $(a_1 + a_2, b_1 + b_2)$ are collinear, show that $a_1 b_2 = a_2 b_1$.

29) Using matrix method to solve the following system of equations
x– 2y -4 =0 -3x + 5y +7 =0.

30) Using matrix method to solve the following system of equations

x+ 2y +z = 7 , x + 3z = 11 2x -3y =1

31) If A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of linear equations x + 2y +z =4
-x + y +z =0, x -3y +z =2

32) Determine the product of $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations x - y +z = 4 , x -2y -2z =9 2x + y +3 z = 1.

33) An amount of Rs 5000 is put in to three investments at the rate of interest of 6% 7% and 8% per annum respectively. The total annual income is Rs 358.If the combined income from the first two investments is Rs 70 more than the income from the third, find the amount of each investment using matrix method.

34) Two schools A and B want to give award their selective students on the values of sincerity, truthfulness and helpfulness. The school A wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs 1600.School B wants to spend Rs 2300 to award its 4,1 and 3 students on the respective values(by giving the same award money to the three values as before). If the total amount of award for one prize on each value is Rs 900 , using matrices. Find the award money for each value.

35) Discuss the continuity of function $f(x) = \begin{cases} \frac{e^x - 1}{\log(1+2x)}, if\ x \neq 0 \\ 7, \quad if\ x = 0 \end{cases}$ at x = 0.

36) Show that $f(x) = \begin{cases} \frac{sin3x}{tan2x}, & if x < 0 \\ 3/2, & if\ x = 0 \\ \frac{\log(1+3x)}{e^{2x}-1}, & if\ x > 0 \end{cases}$ is continuous at x = 0.

37) Determine the values of a, b, c for which the function

$f(x) = \begin{cases} \frac{\sin(a+1)x + sinx}{x}, & if x < 0 \\ C, & if\ x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}}, & if\ x > 0 \end{cases}$ is continuous at x = 0

.38) Determine the values of a and b for which the function

$f(x) = \begin{cases} \frac{1 - sin^3 x}{3cos^2 x}, & if x < \frac{\pi}{2} \\ a, & if\ x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi - 2x)^2}, & if\ x > \frac{\pi}{2} \end{cases}$ is continuous at x = $\frac{\pi}{2}$.

39) Determine the values of k for which the function

$f(x) = \begin{cases} \frac{2^{x+2} - 16}{4^x - 16}, & if\ x \neq 2 \\ k, & if\ x = 2 \end{cases}$ is continuous at x =2.

40) Show that the function $f(x) = |x + 1| + |x - 1| \forall \in R$ is not differentiable at x = -1 and x= 1.

****